

# Low Energy Description of Quantum Gravity and Complementarity

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## Abstract

We propose an explicit framework in which low energy dynamics of quantum gravity is described preserving locality, and yet taking into account the effects that are not captured by the naive global spacetime picture, e.g. those associated with black hole complementarity. Our framework employs a “special relativistic” description of gravity; specifically, gravity is treated as a force measured by the observer tied to the coordinate system associated with a freely falling local Lorentz frame. We explicitly identify regions of spacetime in which low energy local descriptions are applicable as viewed from the freely falling frame; in particular, we identify a surface called the gravitational observer horizon on which the local proper acceleration measured in the observer’s coordinates becomes the cutoff (string) scale. This allows for separating clearly between the “low-energy” local physics and “trans-Planckian” intrinsically quantum gravitational (stringy) physics, and allows for developing clear physical pictures of the origins of various effects. We construct a specific Hilbert space in which the proposed scheme is realized in a simple manner, and classify its elements according to certain horizons they possess. We also discuss implications of our framework on the firewall problem. We conjecture that the complementarity picture may persist due to properties of trans-Planckian physics.

# 1 Introduction

In the past few decades, it has become increasingly clear that quantum theory of gravity will not be built on a simple global spacetime picture of classical general relativity. Quantum mechanics requires a large deviation from the simple global spacetime picture even at long distances—distances much larger than the fundamental scale  $l_*$ , which is expected to be close to the 4-dimensional Planck length  $l_{\text{Pl}} \simeq 1.62 \times 10^{-35}$  m. General relativity must arise as an effective theory—not in the simplest Wilsonian sense—describing observations performed by classical observers.

Historically, the first hint of this has come from studying black holes. The standard local formulation of quantum gravity leads to inconsistency when describing a process in which an object falls into a black hole that eventually evaporates, since it may employ a class of equal time hypersurfaces (called nice slices) on which quantum information is duplicated [1]. In the early 90’s, a remarkable suggestion to avoid this difficulty—called the complementarity picture—has been made [2, 3]: the apparent cloning of the information occurring in black hole physics implies that the internal spacetime and horizon/Hawking radiation degrees of freedom appearing in different, i.e. infalling and distant, descriptions are not independent. This clearly signals a breakdown of the naive global spacetime picture of general relativity, and forces us to develop a new low energy theory of quantum gravity in which locality is preserved (if there exists such a formulation).

In this letter, building on earlier suggestions in Refs. [4, 5], we propose an explicit framework in which low energy dynamics of quantum gravity is described preserving locality, and yet taking into account the effects that are not captured by the naive global spacetime picture. We introduce an explicit coordinate system associated with a freely falling reference frame, which we call the observer-centric coordinate system, that allows for a “special relativistic” description of gravity, i.e. treating gravity *as a force* measured by the observer tied to this coordinate system. This allows us to identify explicitly boundaries of spacetime, which we call the gravitational observer horizon, where the local description of the system breaks down. We propose a specific Hilbert space, which we refer to as the covariant Hilbert space for quantum gravity, in which the proposed scheme is realized in a simple manner. We also discuss how this framework may address the firewall problem [6], i.e. how it allows for keeping the basic hypotheses of complementarity: (1) unitarity of quantum mechanics, (2) the validity of semi-classical descriptions of physics outside the stretched horizon, and (3) the equivalence principle (no “drama” at the horizon for an infalling object), under certain assumptions on microscopic physics at scales above  $M_* \equiv 1/l_*$ .<sup>1</sup> More detailed discussions on the framework described here, including the basic philosophy motivating it and further evidence supporting its validity, will be presented in the upcoming paper [7].

In this letter we limit our discussions to the case of four spacetime dimensions, but the extension

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<sup>1</sup>The fundamental scale  $M_*$  is related to the Planck scale  $M_{\text{Pl}} \equiv 1/l_{\text{Pl}}$  by  $M_{\text{Pl}}^2 \sim NM_*^2$  in four dimensions, where  $N$  is the number of species existing below  $M_*$ .

to other dimensions is straightforward. We take the Schrödinger picture throughout, and we work with a metric signature that is mostly positive.

## 2 Covariant Hilbert Space for Quantum Gravity

Our construction is based on a series of hypotheses. More detailed descriptions, as well as motivations, of these hypotheses will be given in Ref. [7] (see also [4]). Here we simply list them without much elaboration.

We postulate

- (i) A Hamiltonian formalism exists that describes a quantum mechanical system with gravity. Since a system with gravity in general has constraints, we consider the constrained Hamiltonian formalism developed by Dirac [8].
- (ii) There is a way to restrict Hilbert space (e.g. fix intrinsically stringy gauge redundancies) in such a way that dynamics defined on it is local in spacetime at length scales larger than  $l_*$ . In other words, there is a way to formulate a theory such that “intrinsically quantum gravitational” (stringy) effects decouple at distances larger than  $l_*$  (the string scale).
- (iii) The desired local description is obtained by restricting the Hilbert space such that its element represents either an appropriately restricted region of a spacetime hypersurface (when it allows for a spacetime interpretation) or an intrinsically quantum mechanical state (when it does not). In particular, the former can be taken to represent a state of physical degrees of freedom on *a portion of* the past light cone of a fixed reference point  $p_0$ .

A main motivation for the last hypothesis is that it seems to constitute the minimal deviation from the standard general relativistic view of spacetime, needed to address the issue of information cloning in the existence of a horizon. The use of a light cone is also motivated to make the causal structure manifest; the hypersurface represented by a state corresponds to the spacetime region from which a hypothetical observer at  $p_0$  can obtain light ray signals. (The possibility of using a spacelike hypersurface will be mentioned later.)

We argue that the desired description is obtained by suitably dropping some of the constraints needed to reduce the Hilbert space to that of the physical states:

$$\mathcal{P}^\mu(\mathbf{x}) |\Psi\rangle = 0, \tag{1}$$

where  $\mu = 0, \dots, 3$ , and  $\mathbf{x}$  are the coordinates parameterizing a hypersurface on which the states are defined. (For more detailed discussions, see Ref. [7].) Note that  $|\Psi\rangle$  represents a quantum state for the *entire* system, including possible degrees of freedom associated with the boundaries of space, which may be located at infinity. Now, a natural way to define locality is through the

structure of the Hamiltonian. However, if we define the Hamiltonian operator (which is a linear combination of  $\mathcal{P}^\mu(\mathbf{x})$ 's) on Hilbert space  $\mathcal{H}_{\text{phys}}$  spanned by the independent physical states  $|\Psi\rangle$ , then it is simply zero. Furthermore, it is in general not possible to take a basis in  $\mathcal{H}_{\text{phys}}$  such that all of its elements represent well-defined semi-classical spacetimes as they are generically in superposition states. This precludes us from labeling the elements of  $\mathcal{H}_{\text{phys}}$  according to physical configurations in spacetime, since they do not even have well-defined spacetimes.

These issues can be addressed if we consider a Hilbert space larger than  $\mathcal{H}_{\text{phys}}$  by appropriately dropping some of the constraints (which then must be imposed later as the “dynamics” of the system). Specifically, consider a (hypothetical) reference point  $p_0$  at some  $\mathbf{x}$  and a local Lorentz frame elected there. We may then change the basis of constraint operators  $\mathcal{P}^\mu(\mathbf{x})$  (by taking their linear combinations) so that it minimizes the number of constraints corresponding to transformations affecting the local Lorentz frame. This leads to 10 constraint operators,  $H$ ,  $P_i$ ,  $J_{[ij]}$ , and  $K_i$  ( $i = 1, 2, 3$ ), associated with the change of the local Lorentz frame. These operators obey the standard Poincaré algebra. (The set of operators determined in this way is not unique, and each choice corresponds to adopting different, e.g. null or spacelike, quantization.)

We now postulate

- (iv) By dropping the constraints related to the changes of the local Lorentz frame

$$H |\Psi\rangle = P_i |\Psi\rangle = J_{[ij]} |\Psi\rangle = K_i |\Psi\rangle = 0, \quad (2)$$

we obtain a Hilbert space  $\mathcal{H}_{\text{QG}}$  larger than  $\mathcal{H}_{\text{phys}}$ . The elements of  $\mathcal{H}$ —the subspace of  $\mathcal{H}_{\text{QG}}$  allowing for a spacetime interpretation—can then be labeled by physical configurations in spacetime hypersurfaces (together with possible other labels such as spins of particles); in other words, we can take a basis of  $\mathcal{H}$  such that all the basis states have well-defined semi-classical spacetimes. Physics defined on this space is local in the bulk of spacetime.

In particular, we assume that we can take specific linear combinations of the constraint operators  $\mathcal{P}^\mu(\mathbf{x})$  such that the appropriate basis states of  $\mathcal{H}$  represent the configurations of physical degrees of freedom on (portions of) the past light cone of  $p_0$ . We then call the corresponding enlarged Hilbert space  $\mathcal{H}_{\text{QG}}$  the *covariant Hilbert space for quantum gravity*.

The Hamiltonian defined on  $\mathcal{H}_{\text{QG}}$  represents local physics on the past light cone of  $p_0$  within a boundary, which we will explicitly determine below. (Here we are considering each component state, i.e. a basis state in  $\mathcal{H}$  in the basis given in (iv). The full quantum state is in general a superposition of these and other states.) This Hamiltonian is not *manifestly* local, since the constraints associated with the coordinate transformations on the past light cone of  $p_0$  are still imposed on  $\mathcal{H} \subset \mathcal{H}_{\text{QG}}$ . In other words, the elements of  $\mathcal{H}$  represent physical states obtained after solving Einstein's equation on the light cone. To recover a manifest locality of the Hamiltonian, we need to introduce appropriate metric degrees of freedom on the light cone and drop the corresponding

constraints from the definition of the Hilbert space. We assert that the resulting Hamiltonian is then manifestly local in the bulk of spacetime (but not at the boundary). In the rest of the letter, we do not bother with this last step and focus our attention on  $\mathcal{H}_{\text{QG}}$ , which is enough to make *physics* local in the bulk (in the sense that there exists an equivalent, though more redundant, description in which the Hamiltonian takes a manifestly local form).<sup>2</sup>

The Hilbert space  $\mathcal{H}_{\text{QG}}$  is the relevant Hilbert space when we discuss “evolution” of a system with gravity. It is true that a physical state of the *entire* system must obey all the constraints, including those in Eq. (2), and thus satisfies

$$\frac{d}{dt} |\Psi\rangle = 0, \quad (3)$$

i.e.  $|\Psi\rangle$  is static. However, in  $|\Psi\rangle$  we can identify a (small) subsystem as the “clock” degrees of freedom, and rewrite the entanglement of these degrees of freedom—represented e.g. by a set of states  $|i\rangle$ —with the rest of the degrees of freedom—represented e.g. by a set of states  $|\psi_i\rangle$ —in the standard form of Schrödinger time evolution of a state  $|\psi_i\rangle$ , where  $i$  plays the role of time [9]. (In the Minkowski space, we are doing this operation implicitly by identifying boundary degrees of freedom at infinity as the clock degrees of freedom; this is why we can consider time evolution, or  $S$  matrix, in Minkowski space without explicitly being bothered by the clock degrees of freedom.) We may then view  $\mathcal{H}_{\text{QG}}$  as the Hilbert space in which  $|\psi(t)\rangle \equiv |\psi_i\rangle$  evolves unitarily according to the “derived” Hamiltonian, which in general depends on the choice of the clock degrees of freedom.<sup>3</sup> (Note that  $|\psi_i\rangle$  are no longer zero eigenvalue eigenstates of  $H$ ,  $P_i$ ,  $J_{[ij]}$ , or  $K_i$ , in general.) Furthermore, complementarity can be viewed as a relation between different low energy descriptions corresponding to different choices of clocks separated beyond each other’s horizon, which are obtained after a suitable action of  $H$ ,  $P_i$ ,  $J_{[ij]}$ , or  $K_i$  to put the clock in the bulk of spacetime in each description. From this perspective,  $|\Psi\rangle$  serves the role of a generating function from which physical predictions can be derived by identifying the clock degrees of freedom and extracting their entanglement with the rest. For more detailed discussions on this construction, see Ref. [7] (also [4]).

We note that while our framework allows for formally writing down the Hamiltonian applicable at length scales larger than  $l_*$ , this is not directly useful in calculating the effect of dynamical spacetime or the result of a reference frame change, since they depend on unknown dynamics of degrees of freedom at the boundaries of space. This problem may be largely bypassed if we are interested only in a coarse-grained description of the system, by employing a certain correspondence

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<sup>2</sup>The commutation relations among field operators may contain apparent non-local terms associated with null quantization, which arise from the fact that massless particles can propagate along the light cone.

<sup>3</sup>In order for this operation to give well-defined time evolution of  $|\psi_i\rangle$  by an ordered Hamiltonian at a macroscopic level, the state  $|\Psi\rangle$  must be in a special low coarse-grained entropy state, at least in branches relevant for the clock degrees of freedom. In a real cosmological situation, when  $|\Psi\rangle$  represents the entire “multiverse state,” this leads to a set of conditions which the Hamiltonian  $H$  defined on  $\mathcal{H}_{\text{QG}}$  must satisfy [10].

principle which we may call the complementarity hypothesis [11]—we can then use a combination of quantum theory and classical general relativity to obtain a coarse-grained description of the evolution of the system. An advantage of our framework in doing this is that it clearly separates between the “low-energy” local physics and “trans-Planckian” intrinsically quantum gravitational (stringy) physics, so it allows for developing clear physical pictures of the origins of various effects. To obtain a complete dynamical theory, however, we would need to formulate the theory applicable above  $M_*$ —presumably string theory—along the lines described here. This is beyond the scope of the present work.

The structure of the covariant Hilbert space takes the form

$$\mathcal{H}_{\text{QG}} = \mathcal{H} \oplus \mathcal{H}_{\text{sing}}. \quad (4)$$

Here,  $\mathcal{H}$  is spanned by all the possible physical configurations realized on (portions of) the past light cone of  $p_0$  *as viewed from a local Lorentz frame at  $p_0$* , while  $\mathcal{H}_{\text{sing}}$  contains intrinsically quantum mechanical states that do not allow for a spacetime interpretation (the states relevant when  $p_0$  hits a spacetime singularity), where  $\dim \mathcal{H}_{\text{sing}} = \infty$  [4]. How do we define physical configurations “as viewed from a local Lorentz frame at  $p_0$ ”? Where is the boundary of space that determines the relevant portion of the light cone for each element of  $\mathcal{H}$ ? In the next section, we address these questions and provide an explicit prescription to specify elements of  $\mathcal{H}$ . We also discuss a global structure of  $\mathcal{H}$ , based on a certain classification scheme for the elements.

### 3 Defining Boundaries and Classifying the States

We now focus on  $\mathcal{H}$  and identify a spacetime region (in particular, a region on an “equal-time” hypersurface) represented by its element. We discuss how independent quantum states comprising  $\mathcal{H}$  are specified, and classify them into elements of  $\mathcal{H}_{\partial\mathcal{M}}$ ’s, the subsets of  $\mathcal{H}$  labeled by “horizons” possessed by the states.

#### 3.1 Observer-centric coordinates

We first introduce a useful coordinate system to describe our construction. Let us choose a fixed spacetime point  $p_0$  in a fixed spacetime background. We consider that an element of  $\mathcal{H}$  represents physical configurations of dynamical degrees of freedom and their conjugate momenta on the past light cone of  $p_0$ , which we call  $L_{p_0}$ . In general, the elements of  $\mathcal{H}$  are labeled by a set of quantum numbers (i.e. the response to a set of quantum operators), and in Section 3.5 we will discuss how many independent such quantum states exist in full quantum gravity. For now, however, it is sufficient to keep in mind that the state is specified by the response to the operators defined on  $L_{p_0}$ .

Now, consider a timelike geodesic  $p(\tau)$  which passes through  $p_0$  at  $\tau = 0$ :  $p(0) = p_0$ . We take  $\tau$  to be the proper time measured at  $p$ . A set of local Lorentz frames elected along  $p(\tau)$  corresponding to a freely falling frame can then be uniquely determined by specifying spacetime location  $q^\mu$  and proper velocity  $v^i$  of  $p$  at  $\tau = 0$

$$x_{p(0)}^\mu = q^\mu, \quad \left. \frac{dx_{p(\tau)}^i}{d\tau} \right|_{\tau=0} = v^i, \quad (5)$$

as well as 3 Euler angles  $\alpha^{[ij]}$  that determine the orientation of the coordinate axes, where  $i = 1, 2, 3$ . This is because all the axes of the local Lorentz frames along  $p(\tau)$  can be obtained by parallel transporting the axes at  $p(0)$ .

We now introduce angular coordinates  $(\theta, \phi)$  at each  $\tau$  which coincide with the angular variables of the spherical coordinate system of the local Lorentz frame in an infinitesimally small neighborhood of  $p(\tau)$ . We then define the “radial” coordinate  $\lambda$  for fixed  $\tau, \theta, \phi$  as the affine parameter associated with the light ray emitted toward the past from  $p(\tau)$  in the direction of  $(\theta, \phi)$ . The origin and normalization of  $\lambda$  are taken so that the values of  $\lambda$  agree with those of the radial coordinate of the local Lorentz frame in an infinitesimally small neighborhood of  $p(\tau)$ . We perform this procedure in an inextendible spacetime; for example, we do not terminate the light ray at a coordinate singularity. This process allows us to introduce the coordinate system, which we call the *observer-centric coordinate system*. It has 4 coordinates  $\tau, \lambda, \theta$ , and  $\phi$ , depicted schematically in Fig. 1, and provides a reference frame from which physics is described. Note that a hypersurface with constant  $\tau$  corresponds to the past light cone of  $p(\tau)$ , which is a null, rather than spacelike, hypersurface. To describe a state, we need this coordinate system only in an infinitesimally small neighborhood of the  $\tau = 0$  hypersurface. The reason why we need the neighborhood is that some phase space variables involve the  $\tau$  derivative of quantum fields at  $\tau = 0$ .

For a given spacetime, we may convert a coordinate system  $x^\mu$  to the observer-centric one once a local Lorentz frame is elected. For this purpose, we regard  $x^\mu$  to be functions of the observer-centric coordinates,  $x^\mu(\tau, \lambda, \theta, \phi)$ , and derive equations that allow us to solve these functions. Note that the form of these functions depends on the choice of the local Lorentz frame,  $(q^\mu, v^i, \alpha^{[ij]})$ .

### 3.2 Gravitational observer horizon

In general, an element in  $\mathcal{H}$  represents only a portion of  $L_{p_0}$ . Specifically, a past-directed light ray emitted from  $p_0$  hits a point beyond which the semi-classical description of spacetime is not applicable. The collection of these points forms a two-dimensional surface. Here we identify one origin of such surfaces.

Consider a spacetime trajectory of a point with constant  $(\lambda, \theta, \phi)$  in the infinitesimal vicinity

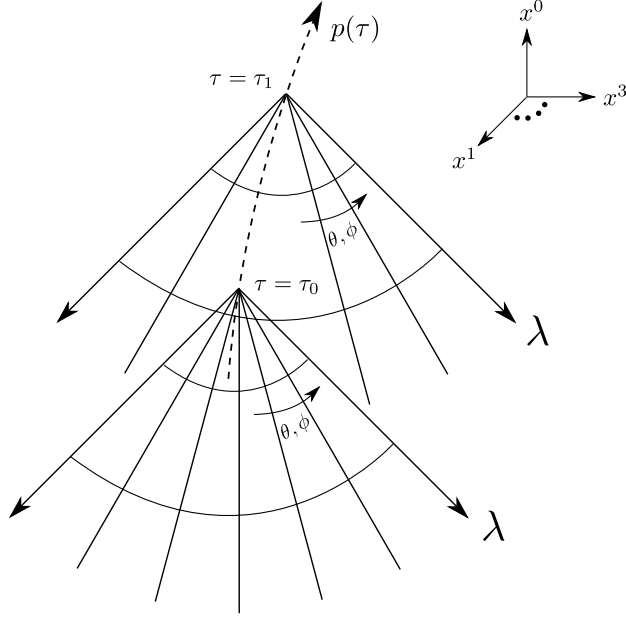


Figure 1: A schematic depiction of the observer-centric coordinate system.

of  $L_{p_0}$ . Its proper velocity is given by

$$u^\mu = \frac{\frac{\partial x^\mu}{\partial \tau}}{\sqrt{-g_{\mu\nu} \frac{\partial x^\mu}{\partial \tau} \frac{\partial x^\nu}{\partial \tau}}}, \quad (6)$$

while the local proper acceleration by

$$a^\mu = u^\nu \nabla_\nu u^\mu. \quad (7)$$

Here,  $a^\mu$  has the dimension of energy in natural unit  $\hbar = c = 1$ , and  $a^\mu = 0$  at  $p_0$  because  $p(\tau)$  is a geodesic. Note that  $u^\mu$  is timelike while  $a^\mu$  is spacelike (or zero) within a horizon  $g_{\tau\tau} = g_{\mu\nu}(\partial x^\mu/\partial \tau)(\partial x^\nu/\partial \tau) = 0$ , where these vectors diverge.

We consider that the semi-classical description of spacetime breaks down when the magnitude of the local proper acceleration vector  $a^\mu$  exceeds the cutoff scale  $M_*$ . Namely, the semi-classical picture is applicable only on a portion of  $L_{p_0}$  in which

$$A \equiv \sqrt{a^\mu a_\mu} \lesssim M_*. \quad (8)$$

This is a natural criterion given that  $a^\mu$  measures acceleration relative to a free-fall. It can be interpreted as the condition that the gravitational acceleration measured from the reference frame—i.e. using the observer-centric coordinates—must be smaller than  $M_*$ . The set of points on  $L_{p_0}$  on



which the above inequality is saturated composes a two-dimensional surface

$$\lambda = \lambda_{\text{obs}}(\theta, \phi), \quad (9)$$

which we call the *gravitational observer horizon*, or the observer horizon for short. We consider that a past-directed light ray emitted from  $p_0$  must be terminated when it hits this surface.

As will be explicitly shown in Ref. [7], in simple spacetimes the local Hawking temperatures on surfaces  $\lambda = \lambda_{\text{obs}}$  actually become of order  $M_*$ , so the semi-classical picture is indeed expected to break down there. In fact, in these spacetimes the observer horizons for approximately static  $p(\tau)$  trajectories are reduced to the stretched horizons defined in Ref. [2]. (In de Sitter space, for example, the observer horizon is located at  $r = 1/H - O(H/M_*^2)$  in the static coordinates when calculated from  $p(\tau)$  staying at the origin. Here,  $H$  is the Hubble constant.) An important point, however, is that unlike the stretched horizon, the definition of the observer horizon does not require knowledge of spacetime outside of  $L_{p_0}$ . This is a desirable feature, as it allows us to construct a state without relying on the information in the spacetime region outside the one represented by the state. We also note that the spacetime location of the observer horizon, as well as the functional form of  $\lambda_{\text{obs}}(\theta, \phi)$ , depends in general on the choice of the reference frame  $(v^i, \alpha^{[ij]})$ . This is another, and in fact crucial, difference of the observer horizon from the stretched horizon defined in a conventional manner.

We consider that each region of the observer horizon holds  $\mathcal{A}/4l_{\text{Pl}}^2$  quantum degrees of freedom, where  $\mathcal{A}$  is the area of the region.<sup>4</sup> This comes from the requirement that the spacetime region “outside” the observer horizon in the global spacetime picture is reproduced by an appropriate reference frame change (complementarity). Our picture is such that the degrees of freedom associated with the “outside spacetime” are entirely in the boundary degrees of freedom on the observer horizon. In fact, as will be discussed in Ref. [7], the number of the boundary degrees of freedom postulated here is sufficient for this purpose because of the holographic principle [12, 13]. (In the case of a black hole viewed from a distant frame, these degrees of freedom are the stretched horizon degrees of freedom.) An element of  $\mathcal{H}$ , therefore, may be said to represent a physical state of the degrees of freedom in *and* on the observer horizon:

$$0 \leq \lambda \leq \lambda_{\text{obs}}(\theta, \phi). \quad (10)$$

Note that the bulk and boundary degrees of freedom will in general be entangled since the horizon forms by a dynamical process. Entanglement between the two will also be necessary to reconstruct the outside region when a relevant reference frame change is made [14].

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<sup>4</sup>The number of degrees of freedom is defined as the natural logarithm of the dimension of the corresponding Hilbert space factor.

### 3.3 Other “ends” of spacetime on $L_{p_0}$

We now discuss other ways in which semi-classical spacetime ceases to exist on  $L_{p_0}$  along a light ray generating it. For this purpose, we first assume that the local proper acceleration is sufficiently small in the relevant region,  $A \ll M_*$ . We argue that there are two ways that the light ray may encounter the “end” of spacetime on  $L_{p_0}$ .

The first possibility is for a light ray to hit a spacetime singularity. Consider a null geodesic representing a light ray emitted from  $p_0$  toward the past in the direction of  $(\theta, \phi)$ . Suppose that the geodesic encounters a spacetime singularity in the sense that it is inextendible beyond some finite value of the affine parameter  $\lambda_{\text{sing}}(\theta, \phi)$  in an inextendible spacetime. In this case, semi-classical spacetime exists only in the region  $\lambda < \lambda_{\text{sing}}(\theta, \phi)$ , and we consider that an element of  $\mathcal{H}$  represents the physical state of the degrees of freedom only in that region.

The other possibility has to do with the behavior of the congruence of past-directed light rays emitted from  $p_0$ . Assuming the null energy condition,  $T_{\mu\nu}v^\mu v^\nu \geq 0$  for all null vectors  $v^\mu$ , the expansion of the light rays  $\Theta$  satisfies [15]

$$\frac{\partial \Theta}{\partial \lambda} + \frac{1}{2}\Theta^2 \leq 0. \quad (11)$$

This implies that the light rays emitted from  $p_0$  converge toward the past, starting from  $\Theta = +\infty$  at  $\lambda = 0_+$ .

Suppose that a light ray reaches a point where  $\Theta = -\infty$  at some finite value of the affine parameter  $\lambda_{\text{conj}}(\theta, \phi)$  (before it hits a spacetime singularity). Such a point is said to be conjugate to  $p_0$ , and signals the failure of the light ray being on the boundary of the past of  $p_0$  [15]. Specifically, there exists a family of timelike causal curves connecting  $p_0$  and a point  $q$  on  $L_{p_0}$  with  $\lambda > \lambda_{\text{conj}}(\theta, \phi)$ . Now, suppose semi-classical spacetime exits beyond  $\lambda_{\text{conj}}(\theta, \phi)$  in our framework. This would contradict the validity of null quantization, which we are assuming throughout. In particular, it would mean that a massive particle sent from  $q$ —which, being on  $L_{p_0}$ , is at an “equal time” as  $p_0$ —can travel backward in time and reach  $p_0$  from the past (as there exists a timelike causal curve connecting  $q$  and  $p_0$ ). We therefore consider that  $\Theta = -\infty$  signals the end of spacetime, and that an element of  $\mathcal{H}$  only represents the region  $\lambda < \lambda_{\text{conj}}(\theta, \phi)$ .

Combining with the possibility of hitting a spacetime singularity discussed above, we conclude that an element of  $\mathcal{H}$  represents a physical state of the degrees of freedom in the region

$$0 \leq \lambda < \lambda_{\text{end}}(\theta, \phi) \equiv \min \{ \lambda_{\text{sing}}(\theta, \phi), \lambda_{\text{conj}}(\theta, \phi) \}, \quad (12)$$

where we have assumed that  $A \ll M_*$  throughout the region. If  $A$  becomes of order  $M_*$  before the light ray reaches a singularity or a conjugate point, i.e.  $\lambda_{\text{obs}}(\theta, \phi) < \lambda_{\text{end}}(\theta, \phi)$ , then spacetime must be terminated there and the boundary degrees of freedom must be attached, according to the discussion in the previous subsection.

We assume that, unlike the observer horizon, the two-dimensional surface determined by  $\lambda = \lambda_{\text{end}}(\theta, \phi)$  does *not* hold boundary degrees of freedom. This corresponds to the hypothesis that the evolution of a state can be determined without any information from the singularity or the region beyond  $\lambda_{\text{conj}}(\theta, \phi)$ , in addition to what is already in the Hamiltonian. For example, the evolution of a big-bang universe is not affected by the “details” of the big-bang singularity that must be specified beyond the Einstein equation. This conjecture seems to be supported in all the (simple) circumstances we have investigated. Further discussions on this and related issues will be given in Ref. [7].

### 3.4 Apparent horizon “pull-back”

We have seen that spacetime on the past light cone of  $p_0$  is extended only until  $\lambda$  reaches  $\lambda_{\text{obs}}$  of Section 3.2 or  $\lambda_{\text{end}}$  of Section 3.3. (Here and below, until Eq. (15), we omit the arguments from the boundary locations, but it should be remembered that they are functions of  $\theta$  and  $\phi$ .) In the former case, the boundary degrees of freedom are attached with the number  $\mathcal{A}/4l_{\text{Pl}}^2$  per area  $\mathcal{A}$ , while in the latter case, none are attached. Here we discuss a description in which this asymmetry of boundary degrees of freedom is dissolved and all the boundaries are treated on equal footing for the purpose of counting degrees of freedom. This description is available if the following condition is satisfied:

$$\lambda_{\text{obs}} \leq \lambda_{\text{sing}} \quad \text{or} \quad \lambda_{\text{conj}} \leq \lambda_{\text{sing}}, \quad (13)$$

i.e. a singularity is screened either by the observer horizon or conjugate point. Indeed, in example spacetimes we have investigated, this condition is always satisfied, although we do not have a proof of it. Below, we assume that Eq. (13) is valid, and disregard a singularity surface.

Let us define the apparent horizon as a surface on which the expansion of the past-directed light rays emitted from  $p_0$  first crosses zero:<sup>5</sup>

$$\Theta = 0 \quad \text{at} \quad \lambda = \lambda_{\text{app}}. \quad (14)$$

This implies that  $\lambda_{\text{app}} < \lambda_{\text{conj}}$ , since  $\Theta$  is a monotonically decreasing function of  $\lambda$ . Now, if  $\lambda_{\text{obs}} < \lambda_{\text{app}}$  for a range of  $(\theta, \phi)$ , then in these directions spacetime ceases to exist at  $\lambda = \lambda_{\text{obs}}$ , where a boundary degree of freedom is located per area  $4l_{\text{Pl}}^2$ . On the other hand, if  $\lambda_{\text{app}} < \lambda_{\text{obs}}$  for a range of  $(\theta, \phi)$ , then there are two cases to consider:

1.  $\lambda_{\text{app}} < \lambda_{\text{conj}} < \lambda_{\text{obs}}$  — In this case, spacetime exists only for  $\lambda < \lambda_{\text{conj}}$ . The covariant entropy bound then implies that the number of physical degrees of freedom in the region  $\lambda > \lambda_{\text{app}}$  is bounded by  $\mathcal{A}/4l_{\text{Pl}}^2$ , where  $\mathcal{A}$  is the area of the relevant portion of the apparent horizon [13, 16].

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<sup>5</sup>This definition is different from that in Ref. [16], where the apparent horizon is defined as a surface on which at least one pair among *four* orthogonal null congruences have zero expansion. Here we only consider two directions along  $L_{p_0}$ .

This suggests that these degrees of freedom may be replaced by  $\mathcal{A}/4l_{\text{Pl}}^2$  boundary degrees of freedom located on the apparent horizon.

2.  $\lambda_{\text{app}} < \lambda_{\text{obs}} < \lambda_{\text{conj}}$  — In this case, physical degrees of freedom outside the apparent horizon consist of the bulk degrees of freedom in  $\lambda_{\text{app}} < \lambda < \lambda_{\text{obs}}$  and the boundary degrees of freedom at  $\lambda = \lambda_{\text{obs}}$ . If the strengthened covariant entropy bound of Ref. [17] applies, then the number of the former is bounded by  $(\mathcal{A} - \mathcal{A}_{\text{obs}})/4l_{\text{Pl}}^2$ , while that of the latter is  $\mathcal{A}_{\text{obs}}/4l_{\text{Pl}}^2$ , where  $\mathcal{A}$  and  $\mathcal{A}_{\text{obs}}$  are the areas of the relevant portions of the apparent and observer horizons, respectively. This suggests that physical degrees of freedom in the region  $\lambda > \lambda_{\text{app}}$  may be replaced by  $\mathcal{A}/4l_{\text{Pl}}^2$  boundary degrees of freedom on the apparent horizon. While the strengthened covariant entropy bound is known to be violated in some extreme cases, we assume that this replacement can always be done in our context.

We thus find that both cases allow for replacing physical degrees of freedom in the region  $\lambda > \lambda_{\text{app}}$  by a quantum degree of freedom per area  $4l_{\text{Pl}}^2$  on the apparent horizon. We call this replacement procedure *apparent horizon pull-back*.

With the apparent horizon pull-back, the structure of the physical region represented by an element of  $\mathcal{H}$  can be stated in the following simple way. Spacetime on  $L_{p_0}$  exists only for

$$0 \leq \lambda \leq \lambda_B(\theta, \phi) \equiv \min \{ \lambda_{\text{obs}}(\theta, \phi), \lambda_{\text{app}}(\theta, \phi) \}. \quad (15)$$

In addition to the degrees of freedom in the bulk of spacetime, the boundary at  $\lambda = \lambda_B(\theta, \phi)$  also holds  $\mathcal{A}/4l_{\text{Pl}}^2$  quantum degrees of freedom, where  $\mathcal{A}$  is the area of the boundary.

### 3.5 Horizon decomposition of $\mathcal{H}$

So far, we have been discussing the structure of spacetime represented by *an* element of  $\mathcal{H}$ . The full Hilbert space  $\mathcal{H}$  consists of the elements representing “all possible” physical configurations in “all possible” spacetimes, as viewed from the reference frame. What do we really mean by that? In other words, what is the structure of  $\mathcal{H}$  concretely?

To address this question, let us adopt the apparent-horizon pulled-back description, discussed in the previous subsection. We now group the elements that have the same boundary  $\partial\mathcal{M}$ , and denote the Hilbert space spanned by these elements by  $\mathcal{H}_{\partial\mathcal{M}}$ .<sup>6</sup> The definition of the boundary being the same is given as follows. Consider the induced metric on the boundary  $\lambda = \lambda_B(\theta, \phi)$  with the arguments being the observer-centric angular variables:

$$h_{XY}(\theta, \phi) = \frac{\partial\lambda_B}{\partial X} \frac{\partial\lambda_B}{\partial Y} g_{\lambda\lambda} + \frac{\partial\lambda_B}{\partial X} g_{\lambda Y} + \frac{\partial\lambda_B}{\partial Y} g_{\lambda X} + g_{XY}, \quad (16)$$

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<sup>6</sup>The  $\mathcal{H}_{\partial\mathcal{M}}$  here is the same as what is denoted by  $\mathcal{H}_{\mathcal{M}}$  in earlier papers Refs. [4, 10, 11].

where  $X, Y = \theta, \phi$ , and  $g_{\lambda\lambda}$ ,  $g_{\lambda X}$ , and  $g_{XY}$  are spacetime metric components in the observer-centric coordinate system, evaluated at  $\tau = 0$  and  $\lambda = \lambda_B(\theta, \phi)$ . We regard two boundaries as the same if the induced metrics on them are *explicitly* identical, i.e. all the  $h_{XY}$ 's ( $X, Y = \theta, \phi$ ) take the identical functional forms with respect to  $\theta, \phi$ .<sup>7</sup>

This definition reflects the fact that our description of physics is “special relativistic” or “as viewed from the reference frame.” For example, a spacetime 2-surface is regarded as different boundaries when described from two different reference frames which are rotated with respect to with each other (unless the surface is spherically symmetric around  $p_0$ ). This implies that depending on the choice of the reference frame, the identical physical configuration in spacetime can belong to different Hilbert subspaces  $\mathcal{H}_{\partial\mathcal{M}}$ . An operator corresponding to rotating the reference frame then transforms an element of a subspace into that of another. Note that here we are talking about a state  $|\psi_i\rangle$  in  $\mathcal{H} \subset \mathcal{H}_{\text{QG}}$ , which may be viewed as representing a physical state relative to clock degrees of freedom. The “full” quantum state (i.e. the multiverse state)  $|\Psi\rangle \subset \mathcal{H}_{\text{phys}}$  obtained after imposing the constraints in Eq. (2) is, of course, invariant under such a rotation (guaranteeing that there is no absolute frame in the universe).

Now, the elements of  $\mathcal{H}_{\partial\mathcal{M}}$  represent all possible physical configurations in all possible spacetimes (or null slices of spacetimes) that share the same boundary  $\partial\mathcal{M}$  as defined above. Let us denote the Hilbert space factors of  $\mathcal{H}_{\partial\mathcal{M}}$  corresponding to the bulk and boundary degrees of freedom by  $\mathcal{H}_{\partial\mathcal{M}, \text{bulk}}$  and  $\mathcal{H}_{\partial\mathcal{M}, B}$ , respectively:

$$\mathcal{H}_{\partial\mathcal{M}} = \mathcal{H}_{\partial\mathcal{M}, \text{bulk}} \otimes \mathcal{H}_{\partial\mathcal{M}, B}, \quad (17)$$

where the direct product structure arises from the locality hypothesis in our framework. According to the covariant entropy bound [13], the dimension of the Hilbert space factor  $\mathcal{H}_{\partial\mathcal{M}, \text{bulk}}$  is given by the area of the boundary  $\mathcal{A}_{\partial\mathcal{M}}$  as  $\dim \mathcal{H}_{\partial\mathcal{M}, \text{bulk}} = \exp(\mathcal{A}_{\partial\mathcal{M}}/4l_{\text{Pl}}^2)$ . On the other hand, by construction the dimension of the boundary factor is  $\dim \mathcal{H}_{\partial\mathcal{M}, B} = \exp(\mathcal{A}_{\partial\mathcal{M}}/4l_{\text{Pl}}^2)$ . Therefore, we find

$$\dim \mathcal{H}_{\partial\mathcal{M}} = \dim \mathcal{H}_{\partial\mathcal{M}, \text{bulk}} \times \dim \mathcal{H}_{\partial\mathcal{M}, B} = \exp\left(\frac{\mathcal{A}_{\partial\mathcal{M}}}{2l_{\text{Pl}}^2}\right). \quad (18)$$

Note that this includes arbitrary fluctuations of spacetimes as well as arbitrary configurations of matter (which are related by Einstein’s equation with each other) that keep the boundary fixed, namely with  $h_{XY}(\theta, \phi)$  held fixed.

The complete spacetime part of the Hilbert space  $\mathcal{H}$  is then given by the direct sum of the Hilbert subspaces  $\mathcal{H}_{\partial\mathcal{M}}$  for different  $\partial\mathcal{M}$ ’s:

$$\mathcal{H} = \bigoplus_{\partial\mathcal{M}} \mathcal{H}_{\partial\mathcal{M}}, \quad (19)$$

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<sup>7</sup>It is not entirely clear if there is no additional condition for the boundaries being the same; for example, we might have to require  $\lambda_B(\theta, \phi)$  to be the same in addition to  $h_{XY}(\theta, \phi)$ . Here we postulate that the identity of  $h_{XY}(\theta, \phi)$  is sufficient, and proceed with it.

where the direct sum runs over  $\partial\mathcal{M} = \{h_{XY}(\theta, \phi)\}$ . We call the expression of this form the *horizon decomposition* of  $\mathcal{H}$ . In general, what  $\partial\mathcal{M}$ 's are included in the decomposition of the complete Hilbert space  $\mathcal{H}$  cannot be determined by the low energy consideration alone. For instance, some spacetimes such as stable (not cosmological) de Sitter space may be unrealistic mathematical idealizations and may not appear in the underlying full quantum theory of gravity. In practice, however, we may include only  $\partial\mathcal{M}$ 's that are relevant to the problem under consideration (the ones relevant for the clock degrees of freedom), and that is sufficient. For discussions of this issue in cosmology, especially in the eternally inflating multiverse, see Ref. [10].

### 3.6 Spacelike quantization

Finally, we discuss briefly if there is a way to use spacelike hypersurfaces, rather than null hypersurfaces, to quantize the system. Such a spacelike quantization would avoid technical subtleties associated with null quantization, for example, non-commutativity of field operators at different points in a same angular direction (see footnote 2).

One possibility is simply to “round” the light cone  $L_{p_0}$  slightly to make an equal-time hypersurface spacelike. We can do this while keeping the boundary  $\partial\mathcal{M}$  fixed. An advantage of this procedure is that the structure of the Hilbert space is unchanged from that in Eqs. (17 – 19). This is because the future-directed ingoing light sheet of  $\partial\mathcal{M}$  (a portion of  $L_{p_0}$  bounded by  $\partial\mathcal{M}$ ) is complete (ending at the caustic at  $p_0$ ), so that the spacelike projection theorem of Ref. [13] applies. In a sense, our null quantization may be viewed as a limit of the spacelike quantization discussed here (although the limit is not completely smooth).

Another possibility is to adopt an “intrinsically spacelike” construction. Specifically, we may follow a similar construction to our covariant Hilbert space using spacelike geodesics attached to the local Lorentz frame at  $p_0$  (e.g. with the affine parameters taken to agree with the radial coordinate in the infinitesimal vicinity of  $p_0$ ), instead of null geodesics (light rays). In particular, we may define acceleration parameter  $A$  and the observer horizon similarly. This construction corresponds to taking different linear combinations of the constraint operators  $\mathcal{P}^\mu(\mathbf{x})$  as  $H$ ,  $P_i$ ,  $J_{[ij]}$ , and  $K_i$  (see discussion in Section 2). The validity of this approach or its relation to the null quantization presented in this letter is not clear. We plan to study this possibility further in Ref. [7].

## 4 Implications on (No) Firewalls

The complementarity picture adopted in our framework is such that the spacetime region outside the observer horizon of  $p_0$ —which appears (only) after a reference frame change—is reproduced entirely from the boundary degrees of freedom located on the horizon. Such a picture has recently been challenged by AMPS [6] (see also [18]), who claim that the smoothness of the horizon (the

equivalence principle) together with the semi-classical nature of spacetime at length scales larger than  $l_*$  is fundamentally incompatible with unitarity of quantum mechanics. Here we argue that this need not be the case, based on the argument given in Ref. [19] (see also [20] for a similar observation). We argue that the complementarity picture, as adopted here and in Ref. [4], may persist because of the (unknown) dynamics of trans-Planckian physics.

Let us recall the “firewall” argument by AMPS. Consider three subsystems of an old black hole system. In a distant frame, these are taken to be (1)  $R$ : early/distant Hawking modes, (2)  $B$ : outgoing modes localized near outside of a (small) patch of the horizon, and (3)  $A$ : a subsystem of the degrees of freedom composing the stretched horizon that is entangled with the modes in  $B$ .<sup>8</sup> In an infalling frame, the *interpretation* of  $A$  (but not of  $R$  or  $B$ ) changes, although it still represents the *same* degrees of freedom (complementarity): (3)’  $A$ : modes inside the horizon that are partner modes of  $B$ . Now, unitarity implies that for an old black hole the entropy of the distant modes decreases as the near modes get out

$$S_{BR} < S_R, \quad (20)$$

where  $S_X$  represents the von Neumann entropy of system  $X$ . On the other hand, the equivalence principle applied to a freely falling observer says

$$S_{AB} = 0, \quad (21)$$

implying  $S_{ABR} = S_R$ . These two relations contradict strong subadditivity of entropy  $S_{AB} + S_{BR} \geq S_B + S_{ABR}$ , since they lead to  $S_B < 0$  if both are true. From this, AMPS conclude that one must abandon either unitarity of a black hole formation/evaporation process (with physics outside the stretched horizon well described by a semi-classical theory) or the equivalence principle.

In Ref. [19], it has been argued that this conclusion is not warranted. A key observation is that unitarity is a statement about an entire quantum state while the equivalence principle is a statement about a classical world—a component/branch of the entire quantum state. Specifically, let us write the quantum state of an old black hole and early radiation as viewed from a distant frame as

$$|\Phi\rangle = \sum_i c_i |R_i\rangle |\text{BH}_i\rangle, \quad (22)$$

where  $|R_i\rangle$  represent states for  $R$  while  $|\text{BH}_i\rangle$  those for the rest of the system, which includes  $B$ ,  $A$ , and the other modes of the black hole than  $A$ . By taking the basis of  $|R_i\rangle$ ’s such that each  $|R_i\rangle$  represents a well-defined phase space configuration of the emitted quanta, each term  $i$  in Eq. (22) represents an already decohered world.

An important assumption in the argument of Ref. [19] is that because of large microscopic degrees of freedom of the black hole, an infalling vacuum may be realized in multiple different

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<sup>8</sup>Following the convention in recent literature, we have changed the symbols denoting various modes. Specifically,  $R$ ,  $B$ , and  $A$  here correspond to  $A$ ,  $B$ , and  $C$  in Refs. [6, 19].

ways at the microscopic level (although general relativity describes all these states as the same infalling vacuum). In particular, these microscopic vacuum states can differ in which subsystem of the black hole is identified as  $A$  (i.e. the partner modes of  $B$ ) and/or how the modes in  $B$  are entangled with those in  $A$ . Let us denote these vacuum states by  $|V_a\rangle$ , which are expected to be nearly orthogonal because of (very) different microscopic configurations. By expanding  $|\text{BH}_i\rangle$  in Eq. (22) as  $|\text{BH}_i\rangle = \sum_a d_{ia} |V_a\rangle$ , the combined black hole and radiation state  $|\Phi\rangle$  can be written as

$$|\Phi\rangle = \sum_{i,a} c_i d_{ia} |R_i\rangle |V_a\rangle \equiv \sum_n \omega_n |\tilde{\Phi}_n\rangle, \quad (23)$$

where  $n \equiv \{i, a\}$  and  $|\tilde{\Phi}_n\rangle \equiv |R_i\rangle |V_a\rangle$ . When a physical object is fallen into the horizon, it is represented as a small excitation in each  $|V_a\rangle$ , so terms with different  $a$  will represent the black hole with the fallen object in different ways at the microscopic level. This implies that terms having different  $n = \{i, a\}$  may be viewed as different, decohered classical worlds.

Now, unitarity of quantum mechanics implies that  $|\Phi\rangle$  satisfies Eq. (20). The strong subadditivity relation then says that Eq. (21) cannot be true. This, however, does *not* mean that general relativity is incorrect. The validity of the equivalence principle requires that the  $AB$  system is maximally entangled *in each classical branch*:

$$\tilde{S}_{AB}^{(n)} = 0, \quad (24)$$

where  $\tilde{S}_{AB}^{(n)}$  is the *branch world entropy* of subsystem  $AB$  in classical world  $n$  [19], i.e. the von Neumann entropy of  $AB$  calculated using the state  $|\tilde{\Phi}_n\rangle$ , rather than  $|\Phi\rangle$ . The point is that relations in Eq. (24) are *not* incompatible with that in Eq. (20) (unless radiation and black hole states factor, i.e.  $d_{ia}$  do not depend on  $i$ ). We may summarize this physics as follows: what is responsible for unitarity of the entire quantum state  $|\Phi\rangle$  is not entanglement between radiation and  $B$  modes, but entanglement between radiation modes and “classical horizon worlds,” in particular the ways the infalling vacuum is encoded in the stretched horizon modes at the microscopic level.

A question then is what is the description of this physics in an infalling frame. According to the implementation here and in Ref. [4], complementarity is realized as a unitary transformation acting on  $\mathcal{H}_{\text{QG}}$  (more fundamentally, a change of the clock degrees of freedom accompanied by the action of an appropriate combination of the Poincaré operators  $H, P_i, J_{[ij]}, K_i$  on relative states  $|\psi_i\rangle$ ). This implies that different  $|\tilde{\Phi}_n\rangle$ ’s in a distant frame must be mapped into different states in an infalling frame. However, in the infalling frame, all these states must represent an infalling vacuum in spacetime where there is no black hole horizon. Where do the necessary degrees of freedom to discriminate these states come from?

We conjecture that they come from boundary degrees of freedom on the observer horizon (more precisely a part of the observer horizon associated with the existence of the black hole) of the infalling reference frame. Remember that the location of the observer horizon depends on



the choice of the reference frame, in particular the spacetime location  $q^\mu$  and velocity  $v^i$  of the reference point  $p_0$ . If  $p_0$  is moving slowly at a location far from the black hole

$$v_p \ll 1, \quad r_p \gg R_S, \quad (25)$$

then the observer horizon associated with the black hole agrees with the conventional stretched horizon, located at

$$r_{\text{obs}} = R_S + O\left(\frac{1}{R_S M_*^2}\right), \quad (26)$$

where  $v_p = |v_i|$ ,  $r_p$  is the radial component of  $q^\mu$  in the Schwarzschild coordinates, and  $R_S$  is the Schwarzschild radius [7].<sup>9</sup> When  $p_0$  enters into the black hole (i.e.  $r_p$  decreases passed  $R_S$ ), the observer horizon recedes so that  $p_0$  does not hit the observer horizon (until it reaches the singularity when  $p_0$  collides with the observer horizon). We find that until  $r_p$  becomes much smaller than  $R_S$ , the area of the observer horizon associated with the black hole is of order  $R_S^2$ , so that there are enough degrees of freedom on the horizon (of order the black hole entropy) to which different  $|\tilde{\Phi}_n\rangle$ 's are mapped into.

In summary, we conjecture that a complementarity transformation provides the following mapping at the microscopic level:

**Distant view:** Different microscopic encodings of the infalling vacuum (and a fallen object) in the degrees of freedom of the black hole stretched horizon (which comprises a part of the observer horizon of the distant reference frame)

$\Longleftrightarrow$

**Infalling view:** The infalling vacuum (and a falling object) in spacetime with different microstates for the observer horizon of the infalling reference frame

In the infalling view, unitarity of the entire quantum state  $|\Phi\rangle$  is ensured by entanglement between early radiation modes and different “observer horizon worlds,” each of which behaves as a different, decohered classical world. The equivalence principle is intact if the structure of the states around  $p_0$  is correctly Minkowski vacuum-like in these classical worlds.

While the argument presented here does not prove complementarity, it shows that complementarity as implemented in the present framework is *not* incompatible for the reason envisioned by AMPS. In particular, if the underlying trans-Planckian dynamics behaves as postulated here (leading to the structure in Eq. (23) and providing the mapping described above), then the complementarity picture—or, in other words, global spacetime picture—can emerge as a property of the trans-Planckian, observer horizon degrees of freedom. Knowing the transformation laws of these

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<sup>9</sup>The part of the observer horizon associated with the black hole can be determined by the values of  $\lambda_{\text{obs}}$ . For example, in Schwarzschild-de Sitter spacetime with  $p_0$  located at  $r_p \ll H^{-1}$  outside the black hole, the values of  $\lambda_{\text{obs}}$  are of  $O(r_p)$  in a small angular region  $\Delta\Omega$  while of  $O(H^{-1})$  in the rest. The observer horizon in  $\Delta\Omega$  is then associated with the black hole, while the rest with the de Sitter horizon. For more detailed discussions, see Ref. [7].

degrees of freedom under the complementarity mapping would then be viewed as a derivation of the generic relativistic, global view of spacetime from the special relativistic, quantum view of gravity presented in this letter. Note, however, that general relativistic global spacetime in which physics occurs locally is an intrinsically classical concept—it arises only after patching various unitarily equivalent states in the same description, and thus vastly overcounts the true degrees of freedom at the quantum level.

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